530.215 MBD, Spring 2009
Solutions for HW#4

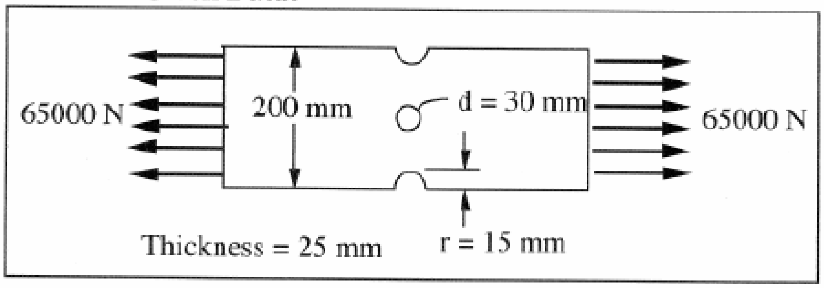
Problem 1

SOLUTION (4.58)

Known: The geometry and the tensile force acting on a specimen are known.

Find: Determine the maximum stress at both the hole and the notch.

Schematic and Given Data:



Assumption: The material is homogeneous and elastic.

Analysis:

1. The nominal stress at the hole and notch is $\sigma_{nom} = \dfrac{65000}{(200 - 60)25} = 18.57$ MPa.
2. For the notch: H/h = 200/170 = 1.2, r/h = 15/170 = 0.09. From Fig (4.39b), K_t = 2.45.
   Therefore, the maximum stress in the notch $\sigma_{Nmax} = 2.45(18.57) = 45.5$ MPa.
3. For the hole: d/b = 30/200 = 0.15. From Fig (4.40b), $K_t = 2.5$.
   Therefore, the maximum stress in the hole $\sigma_{hmax} = 2.5(18.57) = 46.4$ MPa.

Comments:

1. The above stress concentration factor is theoretical based on a theoretical elastic, homogeneous, isotropic material.
2. The value of the maximum stress at the hole is greater than that at the notch.
3. Divide the plate into two notched bars. For each notched bar, $\sigma_{nom} = 18.57$ MPa, H/h = 1.43, r/h = 0.21, $K_t = 1.95$, $\sigma_{max} = 36.2$ MPa.

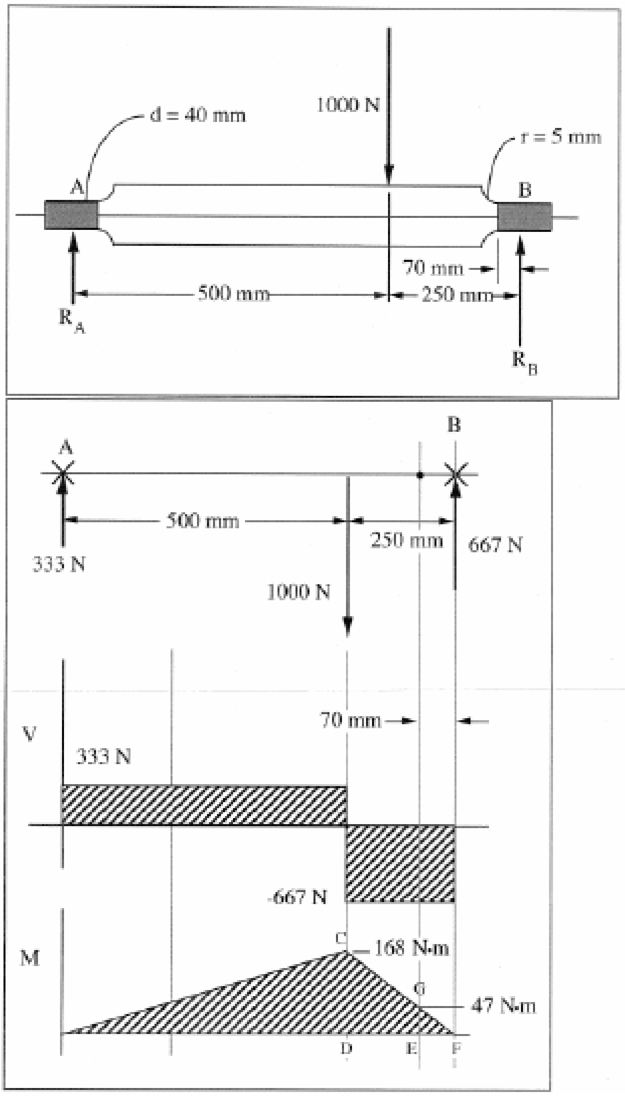
Problem 2

SOLUTION (4.59)

Known: A stepped shaft with known dimensions is supported by bearings and carries a known load.

Find: Determine the maximum stress at the shaft fillet.

Schematic and Given Data:



Assumptions:

1. The shaft remains straight .
2. The material is homogeneous and perfectly elastic.

Analysis:

1. $\Sigma M_B = 0$: Hence $R_A = 333$ N

   $\Sigma F_Y = 0$: $R_A + R_B = 1000$, Therefore $R_B = 667$ N
2. From similar triangle, ΔCDF and ΔGEF,

   GE = 47 N•m. The stress due to bending at the critical shaft fillet is equal to

   $\sigma_{nom} = \dfrac{32M}{\pi d^3} = \dfrac{32(47)}{\pi(0.04)^3} = 7.5$ MPa.
3. r/d for the critical shaft fillet = 5/40 = 0.125

   D/d = 80/40 = 2

   From Fig. (4.35a), $K_t = 1.65$

   Therefore, $\sigma_{max} = \sigma_{nom} K_t = 7.5(1.65) = 12.4$ MPa

Comment: The above stress concentration factor is theoretical based on a theoretical elastic, homogeneous, isotropic material.

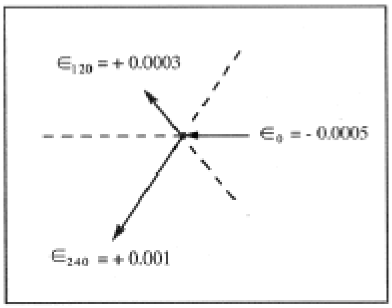
Problem 3

SOLUTION (5.2)

Known: Three readings are obtained from an equiangular strain gage rosette mounted on a free and unloaded surface of a part.

Find: Determine the magnitude of the principal strains and their orientation with respect to the 0° gage. Check the results with a Mohr circle.

Schematic and Given Data:



Assumption: The three known strains are all linear strains.

Analysis:

1. From Eq. (5.1),

$$\varepsilon_{1,2} = \frac{\varepsilon_0+\varepsilon_{120}+\varepsilon_{240}}{3} \pm \sqrt{\frac{(2\varepsilon_0-\varepsilon_{120}-\varepsilon_{240})^2}{9} + \frac{(\varepsilon_{120}-\varepsilon_{240})^2}{3}}$$

$$\varepsilon_{1,2} = \frac{-0.0005+0.0003+0.001}{3} \pm \sqrt{\frac{(-0.001-0.0003-0.001)^2}{9} + \frac{(0.0003-0.001)^2}{3}}$$

$= 0.000267 \pm 0.000867$

Thus, $\varepsilon_1 = 0.00113$ m/m, $\varepsilon_2 = -0.000600$ m/m. ■

2. From Eq. (5.2),

$$\tan 2\alpha = \frac{\sqrt{3}(\varepsilon_{120}-\varepsilon_{240})}{2\varepsilon_0-\varepsilon_{120}-\varepsilon_{240}}$$

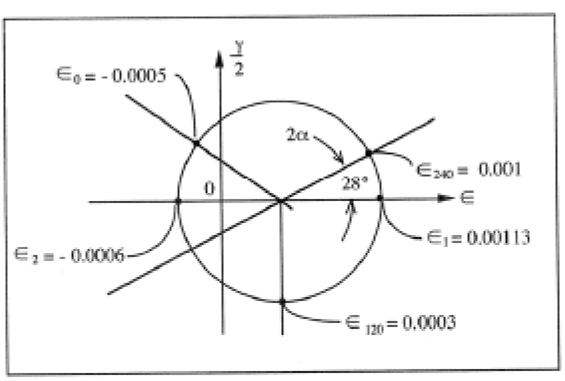
$$\tan 2\alpha = \frac{\sqrt{3}(0.0003-0.001)}{-0.001-0.0003-0.001} = 0.527$$

$2\alpha = 28°, 208°$

So, $\alpha = 14°, 104°$

Use the rule that the higher principal strain always lies within 30° of the algebraically highest of ε_0, ε_{120}, ε_{240}. Then, ε_1 is 74° clockwise from the 0° gage and ε_2 is 164° clockwise from the 0° gage. ■

3.



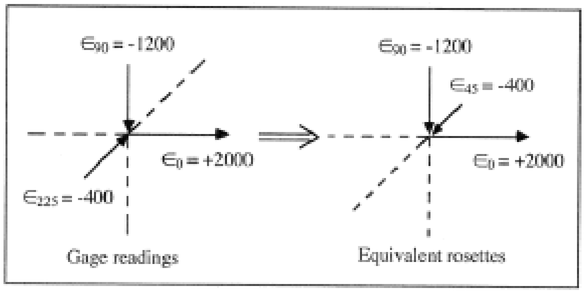
Problem 4

SOLUTION (5.6)

Known: Three readings are obtained from a rectangular strain gage rosette mounted on a free and unloaded surface.

Find: Determine the magnitude of the principal strains and their orientation with respect to the 0° gage. Check the results with a Mohr circle.

Schematic and Given Data:



Assumption: The three known strains are all linear strains.

Analysis:

1. From Eq. (5.3), $\varepsilon_{1,2} = \frac{\varepsilon_0 + \varepsilon_{90}}{2} \pm \sqrt{\frac{(\varepsilon_0 - \varepsilon_{45})^2 + (\varepsilon_{45} - \varepsilon_{90})^2}{2}}$

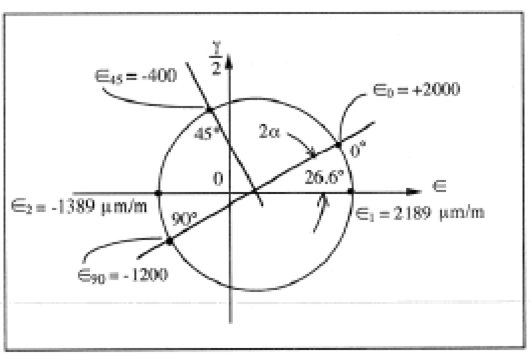
$$= \frac{(2000 - 1200)}{2} \pm \sqrt{\frac{2400^2 + 800^2}{2}} = 400 \pm 1789$$

Thus, $\varepsilon_1 = 2189$ μm/m and $\varepsilon_2 = -1389$ μm/m. ■

2. From Eq. (5.4), $\tan 2\alpha = \frac{\varepsilon_0 - 2\varepsilon_{45} + \varepsilon_{90}}{\varepsilon_0 - \varepsilon_{90}} = \frac{2000 + 800 - 1200}{3200} = 0.5$

So, $\alpha = 13.3°$. Discrimination between the two principal axes can be based on the rule that the algebraically greater principal strain makes an angle of less than 45° with the algebraically larger of strains ε_0 and ε_{90}. Thus, ε_1 is 13.3° clockwise from the +2000 μm/m gage. ■

3.



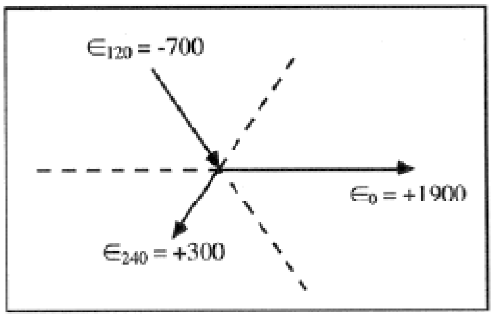
Problem 5

SOLUTION (5.8)

Known: Gage readings are known for an equiangular strain gage rosette mounted on a free and unloaded surface of a part made of steel.

Find: Determine all principal stresses and strains, and draw Mohr circles for both stress and strain.

Schematic and Given Data:



Assumptions:
1. The three known strains are all linear strains.
2. The material remains linearly elastic.

Analysis:
1. From Eq. (5.1),

$$\varepsilon_{1,2} = \frac{\varepsilon_0 + \varepsilon_{120} + \varepsilon_{240}}{3} \pm \sqrt{\frac{(2\varepsilon_0 - \varepsilon_{120} - \varepsilon_{240})^2}{9} + \frac{(\varepsilon_{120} - \varepsilon_{240})^2}{3}}$$

$$\varepsilon_{1,2} = \frac{1900 - 700 + 300}{3} \pm \sqrt{\frac{(3800 + 700 - 300)^2}{9} + \frac{(-700 - 300)^2}{3}} = 500 \pm 1514$$

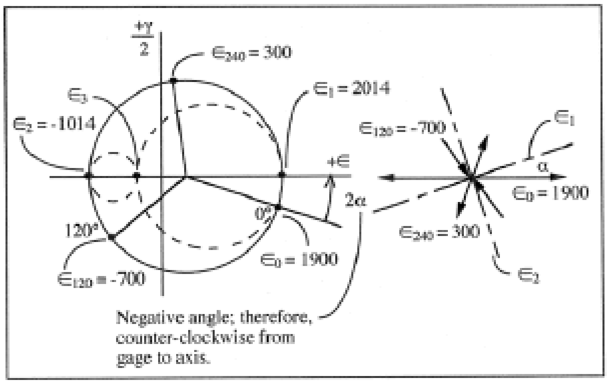
Thus, $\varepsilon_1 = 2014\ \mu\text{m/m}$, $\varepsilon_2 = -1014\ \mu\text{m/m}$. ■

2. From Eq. (5.2),

$$\tan 2\alpha = \frac{\sqrt{3}(\varepsilon_{120} - \varepsilon_{240})}{2\varepsilon_0 - \varepsilon_{120} - \varepsilon_{240}} = \frac{\sqrt{3}(-700 - 300)}{2(1900) + 700 - 300} = -0.412$$

$2\alpha = -22.4^\circ$. So, $\alpha = -11.2^\circ$

3. Use the rule that the higher principal strain always lies within 30° of the algebraically highest of $\varepsilon_0, \varepsilon_{120}, \varepsilon_{240}$. The strain circles are:



4. For steel, E = 207 GPa (Appendix C-1)
$\nu = 0.30$ (Appendix C-1)

Using Eq. (5.5),

$$\sigma_1 = \frac{E}{1 - \nu^2}(\varepsilon_1 + \nu\varepsilon_2) = \frac{207,000}{1 - (0.3)^2}[0.002014 + (0.3)(-0.001014)$$

= 389 MPa

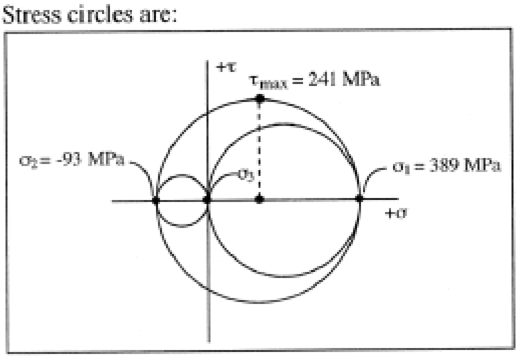
$$\sigma_2 = \frac{E}{1 - \nu^2}(\varepsilon_2 + \nu\varepsilon_1) = \frac{207,000}{1 - (0.3)^2}[-0.001014 + (0.3)(0.002014)]$$

= -93 MPa

$\sigma_3 = 0$

$$\varepsilon_3 = \frac{-\nu}{1 - \nu}(\varepsilon_1 + \varepsilon_2) = \frac{-0.3}{1 - 0.3}(0.002014 - 0.001014) = -429 \ \mu\text{m/m}$$

5. Stress circles are:



Problem 6

10-50 At a point in a structural member subjected to plane stress there are normal and shear stresses on horizontal and vertical planes through the point, as shown in Fig. P10-50. Use Mohr's circle to determine

(a) The principal stresses and the maximum shear stress at the point.

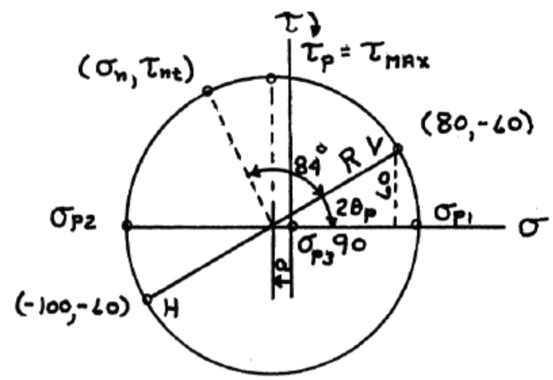
(b) The normal and shear stresses on the inclined plane *AB* shown in the figure.

SOLUTION

The given values for use in drawing Mohr's circle are:

$\sigma_x = +80$ MPa $\quad \sigma_y = -100$ MPa

$\tau_{xy} = -60$ MPa $\quad \sigma_z = \sigma_{p3} = 0$ MPa



$$a = \frac{80 + (-100)}{2} = -10 \text{ MPa}$$

$$R = \sqrt{90^2 + 60^2} = 108.17 \text{ MPa}$$

(a) $\sigma_{p1} = -10 + 108.17 = +98.2$ MPa $= 98.2$ MPa (T) .... **Ans.**

$\sigma_{p2} = -10 - 108.17 = -118.2$ MPa $= 118.2$ MPa (C) .... **Ans.**

$\sigma_{p3} = 0$ MPa .... $\tau_{max} = \tau_p = R = 108.2$ MPa .... **Ans.**

(b) $\sigma_n = -10 - 108.17\cos 62.31° = -60.3$ MPa $= 60.3$ MPa (C) .... **Ans.**

$\tau_{nt} = 108.17\sin 62.31° = 95.8$ MPa ↻ $= -95.8$ MPa .... **Ans.**

Problem 7

10-114 A cylindrical pressure vessel is fabricated by butt-welding 20-mm plate with a spiral seam, as shown in Fig. P10-114. The pressure in the tank is 2800 kPa. Determine

(a) The normal stress perpendicular to the weld.

(b) The shearing stress parallel to the weld.

(c) The maximum shearing stress at a point on the outside surface of the vessel.

(d) The maximum shearing stress at a point on the inside surface of the vessel.

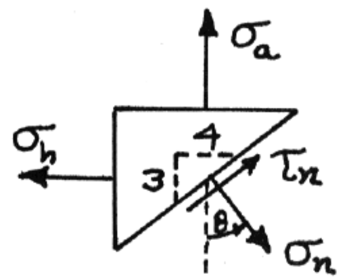
SOLUTION

$$\sigma_a = \frac{pr}{2t} = \frac{(2800\times10^3)(0.600)}{2(0.020)}$$

$$= 42.00\times10^6 \text{ N/m}^2 = 42.00 \text{ MPa}$$

$$\sigma_h = \frac{pr}{t} = \frac{(2800\times10^3)(0.600)}{(0.020)}$$

$$= 84.00\times10^6 \text{ N/m}^2 = 84.00 \text{ MPa}$$



$$\theta = \tan^{-1}(3/4) = 36.870°$$

(a) $$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= 42.00\cos^2(36.870°) + 84.00\sin^2(36.870°) + 0 = 57.1 \text{ MPa}$$ .... **Ans.**

(b) $$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$= -(42.00 - 84.00)\sin(36.870°)\cos(36.870°) + 0 = 20.2 \text{ MPa}$$ .... **Ans.**

(c) $$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\sigma_h - 0}{2} = \frac{84.00 - 0}{2} = 42.0 \text{ MPa}$$ .... **Ans.**

(d) $$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\sigma_h - (-p)}{2} = \frac{84.00 - (-2.800)}{2} = 43.4 \text{ MPa}$$ .... **Ans.**

Problem 8

10-116 A spherical pressure vessel 3 m in diameter is being designed to withstand a maximum internal pressure of 500 kPa. The material being used in its construction has an allowable stress of 105 MPa, a modulus of elasticity of 210 GPa, and a Poisson's ratio of 0.20. Determine

(a) The minimum satisfactory wall thickness.

(b) The circumferential normal strain at maximum pressure when the wall thickness of part (a) is used.

(c) The change in diameter of the pressure vessel at maximum pressure when the wall thickness of part (a) is used.

SOLUTION

(a) $$\sigma = \frac{pr}{2t} = \frac{(0.500)(1.5-t)}{2t} = 105 \text{ MPa}$$

$t = 3.571\times10^{-3} \text{ m} \cong 3.57 \text{ mm}$ .... **Ans.**

(b) $\varepsilon = \dfrac{\sigma - \nu\sigma}{E} = \dfrac{105 - 0.30(105)}{210\times10^{3}} = 350\times10^{-6} \text{ m/m} = 350\ \mu\text{m/m}$ .... **Ans.**

(c) $$\Delta C = \varepsilon C = \varepsilon\pi D = \left(350\times10^{-6}\right)(\pi)(3) = 3300\times10^{-6} \text{ m}$$

$$C + \Delta C = \pi\left(D + \Delta D\right) = C + \pi\Delta D$$

$\Delta D = \dfrac{\Delta C}{\pi} = \dfrac{3300\times10^{-6}}{\pi} = 1050.4\times10^{-6} \text{ m} \cong 1.050 \text{ mm}$ .... **Ans.**