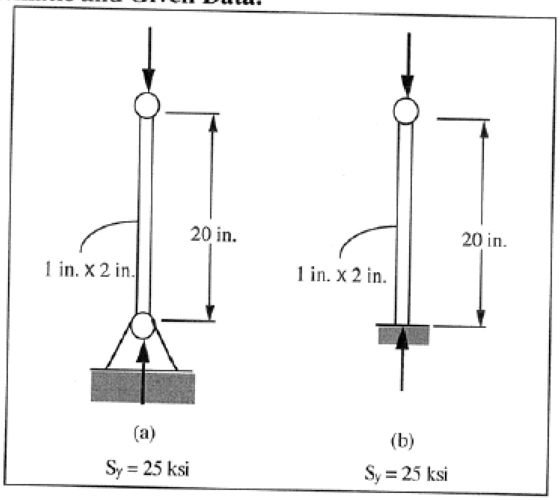
530.215 MBD, Spring 2009
Solutions for HW#6
Problem 1

Schematic and Given Data:



Assumptions:

1. The bar does not fail under compression.
2. The bar is straight.
3. The minimum AISC recommended value for L_e should be used in the analysis.

Analysis:

(a) From Appendix B-1, the least radius of gyration

$$\rho = 0.289\ h = 0.289\ (1) = 0.289 \text{ in.}$$

$$\frac{L_e}{\rho} = \frac{20}{0.289} = 69.2$$

From Eq. (5.13), Euler and Johnson are tangent at:

$$\frac{L_e}{\rho} = \left(\frac{2\pi^2 E}{S_y}\right)^{\frac{1}{2}} \text{ where } E = 30 \times 10^6 \text{ psi (Appendix C-1)}$$

$$L_e/\rho = \left[\frac{2\pi^2(30 \times 10^6)}{25 \times 10^3}\right]^{\frac{1}{2}} = 154$$

Johnson's Eq. (5.12) applies:

$$S_{cr} = S_y - \frac{S_y{}^2}{4\pi^2 E}\left(\frac{L_e}{\rho}\right)^2$$

$$S_{cr} = 25{,}000 - \frac{25{,}000^2}{4\pi^2(30 \times 10^6)}(69.2)^2 = 22{,}470 \text{ psi}$$

$$P_{cr} = S_{cr}A = 22{,}470\ (1)(2) = 44{,}490 \text{ lb.}$$

With a safety factor of 4, P = 11,235 lb

(b) From Fig. 5.25(e), assume $L_e = (2.1)L = 42$ in.

$$\frac{L_e}{\rho} = \frac{42}{0.289} = 145.33$$

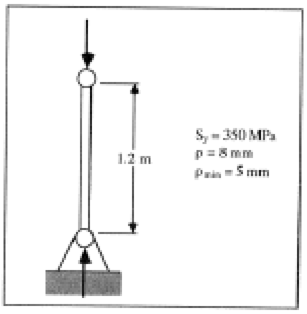
Eq. (5.12) still applies:

$$S_{cr} = 25{,}000 - \frac{25{,}000^2}{4\pi^2(30 \times 10^6)}(145.33)^2 = 13{,}854 \text{ psi}$$

P_{cr} = 13,854(1)(2) = 27,708 lb
With a safety factor of 4, P = 6927 lb

Problem 2

Schematic and Given Data:



Assumptions:
1. The steel angle iron does not fail under compression.
2. The ends are pinned.
3. The steel angle is straight.

Analysis:
1. Using the minimum radius of gyration,

$$\frac{L_e}{\rho} = \frac{1200}{5} = 240$$

Euler/Johnson tangent point [Eq. (5.13)] is at

$$\frac{L_e}{\rho} = \left(\frac{2\pi^2 E}{S_y}\right)^{\frac{1}{2}} = \left[\frac{2\pi^2(207)}{0.350}\right]^{\frac{1}{2}} = 108.05$$

Hence, the Euler Eq. (5.11) applies:

$$S_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L_e/\rho)^2}$$

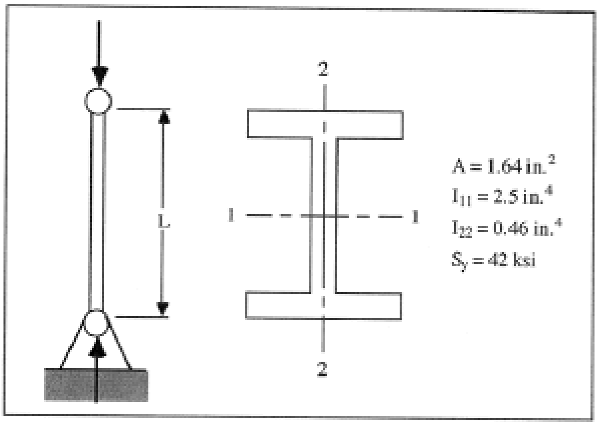
where $E = 207 \times 10^9$ Pa (Appendix C-1)

$$S_{cr} = \frac{\pi^2(207 \times 10^3 \text{ MPa})}{(240)^2} = 35.47 \text{ MPa}$$

Since the cross-sectional area is not given, the load capacity with SF = 3 can only be given as: $P = \frac{35.5\ A}{3}$; P = 11.8 A where"P" is in Newtons and "A" in mm^2. ■

Problem 3

Schematic and Given Data:



Assumptions:
1. The I-beam does not fail under compression.
2. The I-beam is straight.

Analysis: Since $I = A\rho^2$, $\rho_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{0.46}{1.64}} = 0.53$ in.

Euler-Johnson tangent point [Eq. (5.13)] is at $\frac{L_e}{\rho} = \left(\frac{2\pi^2 E}{S_y}\right)^{\frac{1}{2}}$ where $E = 30 \times 10^6$ psi

(Appendix C-1). $\frac{L_e}{\rho} = \left[\frac{2\pi^2(30 \times 10^3)}{42}\right]^{\frac{1}{2}} = 119$

(a) $\frac{L_e}{\rho} = \frac{10}{0.53} = 18.9$. Johnson equation [Eq. (5.12)] applies:

$$S_{cr} = S_y - \frac{S_y^2}{4\pi^2 E}\left(\frac{L_e}{\rho}\right)^2 = 42 - \frac{(42)^2}{4\pi^2(30{,}000)}(18.9)^2 = 41.5 \text{ ksi}$$

with SF = 3, P = (41,500 psi)(1.64 in.²)/3 = 22,670 lb ■
(Note that, in this case, column action is almost negligible, and $S_{cr} \approx S_y$)

(b) $\frac{L_e}{\rho} = \frac{50}{0.53} = 94.34$. Johnson equation still applies:

$$S_{cr} = 42 - \frac{(42)^2}{4\pi^2(30{,}000)}(94.34)^2 = 28.7 \text{ ksi}$$

P = (28,700) (1.64)/3 = 15,710 lb

(c) $\frac{L_e}{\rho} = \frac{100}{0.53} = 188.68$. Euler Eq. (5.11) applies:

$$S_{cr} = \frac{\pi^2 E}{(L_e/\rho)^2} = \frac{\pi^2(30{,}000)}{(188.68)^2} = 8.32 \text{ ksi}$$

P = (8320)(1.64)/3 = 4550 lb

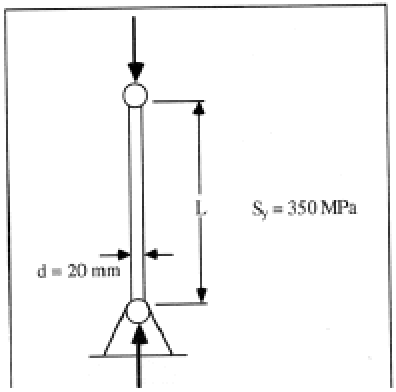
(d) $\frac{L_e}{\rho} = \frac{200}{0.53} = 377.36$. Eq. (5.11) applies:

$$S_{cr} = \frac{\pi^2(30{,}000)}{(377.36)^2} = 2.08 \text{ ksi}$$

P = (2080)(1.64)/3 = 1140 lb

Problem 4

Schematic and Given Data:



Assumptions:
1. The rod does not fail under compression.
2. The rod is straight.

Analysis: From Appendix B-1, $\rho = d/4 = 20/4 = 5$ mm

(a) Assuming Johnson Eq. (5.12) applies:

$$S_{cr} = S_y - \frac{S_y^2}{4\pi^2 E}\left(\frac{L_e}{\rho}\right)^2$$

where E = 207 GPa (Appendix C-1).

$$\sigma_{cr} = (350\text{ MPa})(0.9) = 315\text{ MPa} = 350 - \frac{(350)^2}{4\pi^2(207{,}000)}\left(\frac{L_e}{\rho}\right)^2$$

$$35\text{ MPa} = \frac{(350)^2}{4\pi^2(207{,}000)}\left(\frac{L_e}{\rho}\right)^2. \quad \text{Therefore, } \frac{L_e}{\rho} = 48.32$$

From Eq. (5.13), Euler/Johnson tangent is at

$$\frac{L_e}{\rho} = \left(\frac{2\pi^2 E}{S_y}\right)^{\frac{1}{2}} = \left[\frac{2\pi^2(207{,}000)}{350}\right]^{\frac{1}{2}} = 108$$

Hence, Johnson does indeed apply and $L_e = L = 48.32(5) = 241.6$ mm

(b) Assume Euler Eq. (5.11) applies:

$$S_{cr} = \frac{\pi^2 E}{(L_e/\rho)^2}$$

$$\sigma_{cr} = (350\text{ MPa})(0.5) = 175\text{ MPa} = \frac{\pi^2(207{,}000)}{(L_e/\rho)^2}$$

Therefore, $\frac{L_e}{\rho} = 108$

Hence, both Johnson and Euler apply, and $L_e = L = 108(5) = 540$ mm

(c) The Euler equation will definitely apply.

$$\sigma_{cr} = (350)(0.1) = 35\text{ MPa} = \frac{\pi^2(207{,}000)}{(L_e/\rho)^2} \Rightarrow \frac{L_e}{\rho} = 241$$

$L_e = L = 241(5) = 1205$ mm

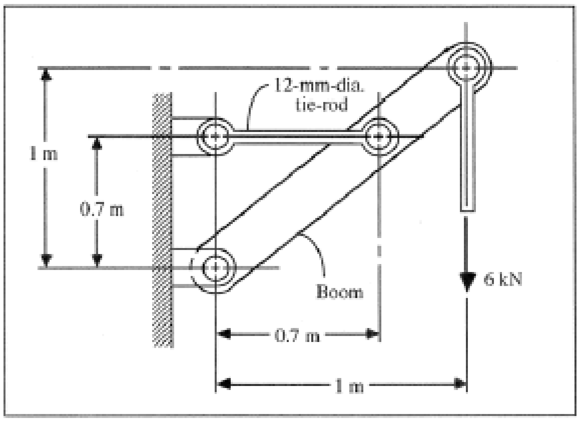
(d) The Euler equation will apply.

$$\sigma_{cr} = (350)(0.02) = 7\text{ MPa} = \frac{\pi^2(207{,}000)}{(L_e/\rho)^2} \Rightarrow \frac{L_e}{\rho} = 540$$

$L_e = L = 540(5) = 2700$ mm

Problem 5

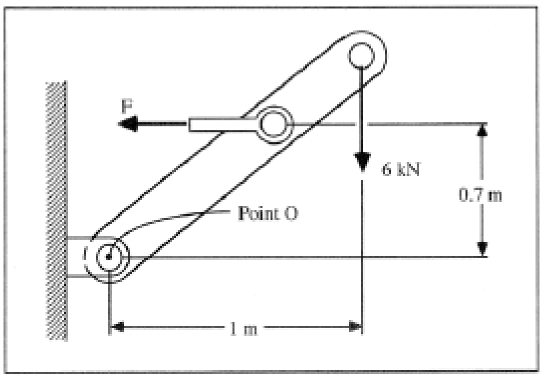
Schematic and Given Data:



Assumption: The tie-rod is straight.

Analysis:

(a)



$$\Sigma M_o = 0:\ F = \frac{6\ \text{kN}(1\ \text{m})}{0.7\ \text{m}} = 8.57\ \text{kN}$$

Tensile stress in the tie rod is $\sigma = \frac{P}{A} = \frac{8570}{36\ \pi\ \text{mm}^2} = 75.8\ \text{MPa}$

$$SF = \frac{400}{75.8} = 5.3$$

■

(b) $\sigma = 75.8$ MPa in compression

From Appendix B-1, $\rho = \frac{d}{4} = 3$ mm, $L = L_e = 700$ mm $\Rightarrow \frac{L_e}{\rho} = 233.3$

Euler-Johnson tangent point [Eq. (5.13)] is at

$$\frac{L_e}{\rho} = \left(\frac{2\pi^2 E}{S_y}\right)^{\frac{1}{2}} = \left[\frac{2\pi^2(207 \times 10^3)}{400}\right]^{\frac{1}{2}} = 101$$

where $E = 207 \times 10^9$ Pa (Appendix C-1).

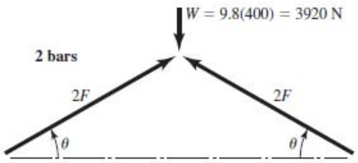
Hence, Euler Eq. (5.11) applies:

$$S_{cr} = \frac{\pi^2 E}{(L_e/\rho)^2} = \frac{\pi^2(207 \times 10^3)}{(233.3)^2} = 37.5\ \text{MPa}$$

Since 75.8 > 37.5, the rod will fail in buckling. ■

(c) Long, slender rods in compression can carry only a small fraction of the load they can carry in tension.

Problem 6.



$$4F\sin\theta = 3920$$

$$F = \frac{3920}{4\sin\theta}$$

In range of operation, F is maximum when $\theta = 15^\circ$

$$F_{\max} = \frac{3920}{4\sin 15} = 3786 \text{ N per bar}$$

$$P_{cr} = n_d F_{\max} = 2.5(3786) = 9465 \text{ N}$$

$l = 300$ mm, $h = 25$ mm

Try $b = 5$ mm: out of plane $k = (5/\sqrt{12}) = 1.443$ mm

$$\frac{l}{k} = \frac{300}{1.443} = 207.8$$

$$\left(\frac{l}{k}\right)_1 = \left[\frac{(2\pi^2)(1.4)(207)(10^9)}{380(10^6)}\right]^{1/2} = 123 \quad \therefore \text{ use Euler}$$

$$P_{cr} = (25)(5)\frac{(1.4\pi^2)(207)(10^3)}{(207.8)^2} = 8280 \text{ N}$$

Try: 5.5 mm: $k = 5.5/\sqrt{12} = 1.588$ mm

$$\frac{l}{k} = \frac{300}{1.588} = 189$$

$$P_{cr} = 25(5.5)\frac{(1.4\pi^2)(207)(10^3)}{189^2} = 11\,010 \text{ N}$$

Use 25 × 5.5 mm bars *Ans.* The factor of safety is thus

$$n = \frac{11\,010}{3786} = 2.91 \quad \textit{Ans.}$$