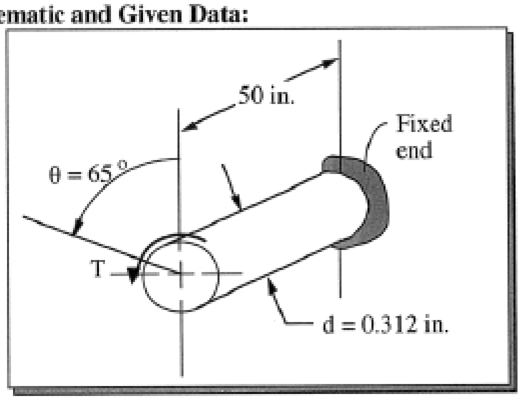
Solutions: HW#10
Problem 1

SOLUTION (12.1)

Known: A steel torsion bar spring is shown in Figure 12.1a. One end of the bar angularly rotates 65° relative to the other end. The length of the torsion bar portion is 50 in. and the diameter is 0.312 in.

Find: Calculate the energy stored in a steel torsion bar. Also, calculate the maximum shear stress.

Schematic and Given Data:



Assumptions:
1. The bar remains straight and the torque is applied about the longitudinal axis.
2. The material is homogeneous and perfectly elastic within the stress range involved.

Analysis:
1. From Table 5.1, for a torsional case (Case 2):

   $\theta = \frac{TL}{JG}$, therefore, $T = \frac{\theta JG}{L}$

2. $\theta = (65/180)\pi = 1.1345$ rad; and $J = \pi d^4/32 = \pi(0.312)^4/32 = 0.00093$ in.^{4}
   From (Appendix C-1), $G = 11.5 \times 10^6$ psi

3. Therefore, $T = \frac{1.1345 \times 0.00093 \times 11.5 \times 10^6}{50} = 242.66$ lb in.

4. The internal energy stored in the spring is equal to the work done to angularly rotate the spring, i. e., $U = (1/2)T\theta = 0.5(242.66 \text{ lb in.})(1.1345 \text{ rad}) = 137.65$ in.-lb. ■

5. From Eq. (4.4), for a solid round rod,

   $\tau = 16T/\pi d^3 = \frac{(16)(242.66)}{\pi(0.312)^3} = 40.69$ ksi

Problem 2

SOLUTION (12.5)

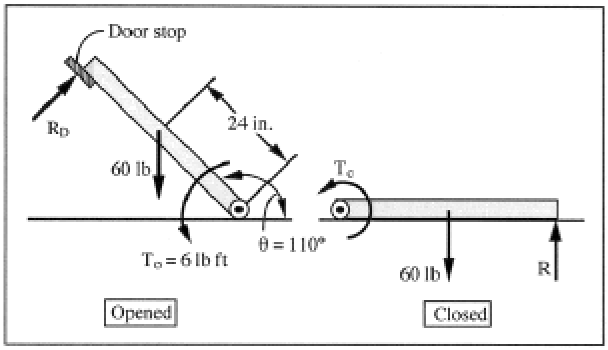
Known: A torsion bar spring serves as a counterbalance for a trap door with a given weight. The maximum allowable torsional stress for the spring is 50 ksi.

Find:

(a) Determine the length and diameter of a solid steel torsion bar that would counterbalance 80% of the door weight when closed, and provide a 6 lb-ft torque holding the door against the stop.

(b) Make a graph showing gravity torque, spring torque, and net torque all plotted against door opening angle.

Schematic and Given Data:



Assumptions:
1. The bar is straight and the torque is applied about the longitudinal axis.
2. The material is homogeneous and perfectly elastic within the stress range involved.
3. The cross section considered is sufficiently remote from points of load application and from stress raisers.

Analysis:

1. From Eq. (4.4), for a solid round rod $\tau = \dfrac{16T}{\pi d^3}$ or $d = \sqrt[3]{16T/\pi\tau}$

   When the door is closed, the bar will counterbalance 80% of the door weight.
   Thus, $T_c = (0.80)(60\text{ lb})(24\text{ in.}) = 1152$ lb in.
   Since a maximum allowable torsional stress is 50 ksi,

   $d = \sqrt[3]{16(1152)/\pi(50{,}000)} = 0.49$ in. ■

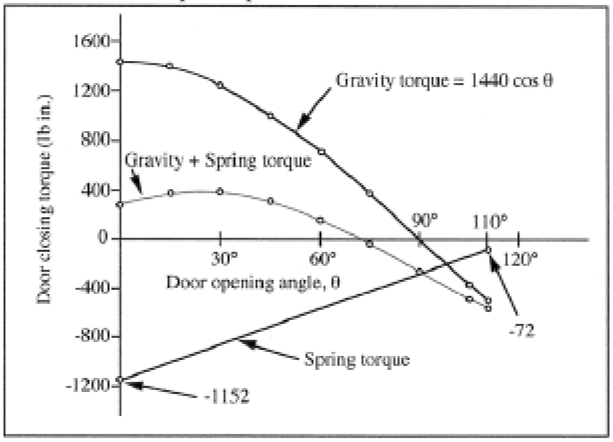
2. From Table 5.1, for a torsional case (Case 2) $\theta = \dfrac{TL}{JG}$ and $K = \dfrac{T}{\theta} = \dfrac{JG}{L}$

   where $J = \dfrac{\pi d^4}{32} = 0.005660\ \text{in.}^4$

   $G = 11.5 \times 10^6$ psi (Appendix C-1)

   $K = \dfrac{\Delta T}{\Delta\theta} = \dfrac{T_c - T_o}{\Delta\theta} = \dfrac{(1152 - 72)\text{ lb in.}}{1.92\text{ rad}} = 562.5$ lb in./rad

3. Let clockwise torque be positive.



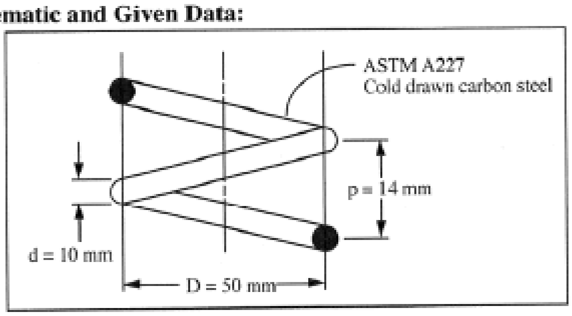
Problem 3

SOLUTION (12.12)

Known: A helical coil spring with given D and d is wound with a known pitch value. The material is ASTM A227 cold drawn carbon steel.

Find: If the spring is compressed solid, would you expect it to return to its original free-length when the force is removed?

Schematic and Given Data:



Assumptions:

1. There are no unfavorable residual stresses.
2. Both end plates are in contact with nearly a full turn of wire.
3. The end plate loads coincide with the spring axis.

Analysis:

1. Force to compress spring solid can be calculated by using Eq. (12.7).

$$F = \frac{d^4 G\delta}{8D^3 N}$$

where $\delta/N = p - d = 14 - 10 = 4$ mm

$G = 79 \times 10^9$ Pa (Appendix C-1)

$$F = \frac{(10 \times 10^{-3})^4 (79 \times 10^9)(4 \times 10^{-3})}{8(50 \times 10^{-3})^3} = 3160 \text{ N}$$ ■

2. The corresponding stress can be calculated by using Eq. (12.6).

$$\tau = \frac{8FD}{\pi d^3} K_s$$

for $C = D/d = 50/10 = 5$

$K_s = 1.1$ (Fig. 12.4)

$$\tau = \frac{8(3160)(50 \times 10^{-3})}{\pi(10 \times 10^{-3})^3}(1.1) = 442.6 \text{ MPa}$$

3. From Eq. (12.9), $\tau_S \le 0.45\, S_u$
   From Fig. 12.7, $S_u \approx 1300$ MPa
   Thus, $0.45\, S_u = 585$ MPa
4. Since 442.6 MPa < 585 MPa, no set should occur; therefore, spring should return to original length. ■

Comment: Even considering the curvature (stress concentration) factor of the inner surface by using $K_w = 1.3$, the inner surface stress is only (1.3)(442.6) = 575.4 MPa which is still less than 585 MPa.

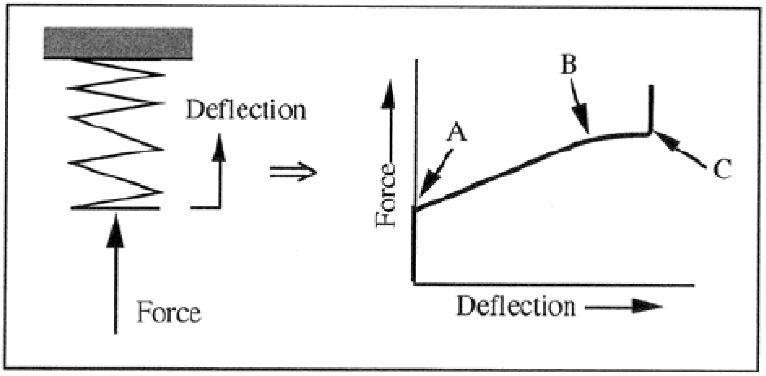
Problem 4

SOLUTION (12.14)

Known: A coiled compression spring is loaded against a support. A plot of the resulting force-deflection curve is shown.

Find: Briefly state the reasons the curve changes at points A, B, and C.

Schematic and Given Data:



Assumptions:
1. There are no unfavorable residual stresses.
2. Both end plates are in contact with nearly a full turn of wire.
3. The end plate loads coincide with the spring axis.

Analysis:
1. At A, the force overcomes the spring preload, causing deflection to begin.
2. At B, significant yielding of the spring begins. (In most instances, this would indicate an improperly designed spring.
3. At C, the spring closes solid.

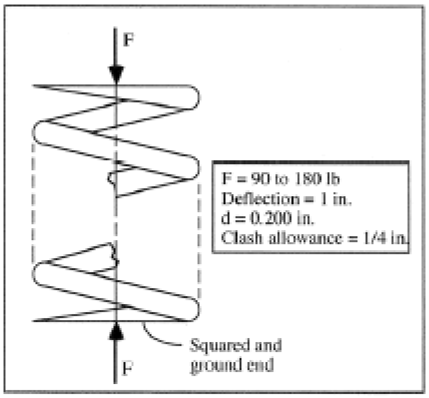
Problem 5

SOLUTION (12.28)

Known: A coil spring with squared and ground ends is to operate with a load that fluctuates between 90 and 180 lb, during which the deflection is to vary by 1 in. Use a steel spring wire having d = 0.200 in. and fatigue strength properties as shown on Fig. 12.16 for shot-peened wire. Presetting, and a clash allowance of 1/4 in. are to be used. Residual stresses due to presetting are not to be taken into account.

Find: Determine appropriate values for N, D, and L_f.

Schematic and Given Data:



Assumptions:
1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.

Analysis--Case A--with shotpeening:
1. From Fig. 12.16, for $\tau_{max}/\tau_{min} = 2$, $\tau_{max} = 800$ MPa = 116 ksi
2. From Eq. (12.5), $\tau = \frac{8F}{\pi d^2} CK_w$ or $CK_w = \frac{\pi d^2 \tau_{max}}{8F_{max}}$

   $$CK_w = \frac{\pi(0.2)^2(116{,}000)}{8(180)} = 10.12$$
3. From Fig.12.4, C = 8.5

   D = Cd = 8.5(0.2) = 1.70 in. ■
4. k = F/δ = 90/1 = 90 lb/in.
5. From Eq. (12.8), $k = \frac{dG}{8NC^3}$ or $N = \frac{dG}{8kC^3}$

   where G = 11.5 × 10⁶ ksi (Appendix C-1)

   $$N = \frac{(0.2)(11.5 \times 10^6)}{8(90)(8.5)^3} = 5.2$$ ■

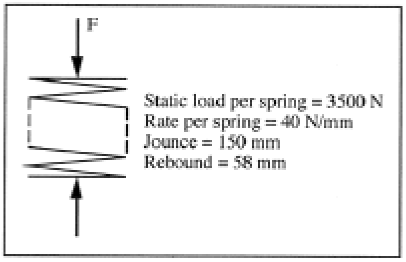
Problem 6

SOLUTION (12.36)

Known: The specifications for a coil spring are known. Infinite fatigue life is required, using a safety factor of 1.3 applied to the maximum load only. The fatigue strength can be represented by a line between $\tau_{max} = 600$, $\tau_{min} = 0$ and $\tau_{max} = \tau_{min} = 900$ MPa.

Find: Determine a suitable combination of d, D, and N.

Schematic and Given Data:



Assumptions:
1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.

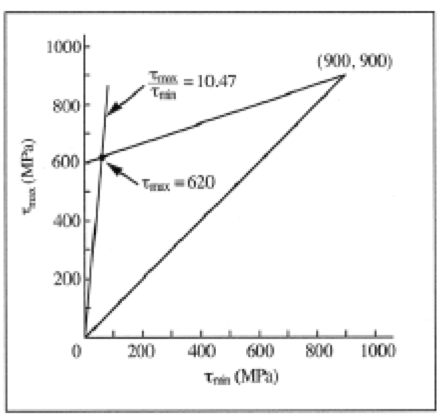
Analysis:
1. First, choose a reasonable value of C, say C = 8. Thus, from Fig. 12.4 $K_w = 1.18$.
2. At the "design overload":
   Using Eq. (12.5),

$$\tau_{max} = \frac{8F_{max}}{\pi d^2} CK_w = \frac{8[(3500 + (40)(150))1.3]}{\pi d^2}(8)(1.18) = 296{,}879/d^2$$

$$\tau_{min} = \frac{8F_{min}}{\pi d^2} CK_w = \frac{8[(3500 - (40)(58)]}{\pi d^2}(8)(1.18) = 28{,}366/d^2$$

$\tau_{max}/\tau_{min} = 296{,}879/28{,}366 = 10.47$

3.



4. From graph, $\tau_{max} = 620$ MPa.

$$620 = \frac{296,879}{d^2} \text{ or } d = 21.88 \text{ mm}^*$$ ■

5. $D = Cd = 8(21.88) = 175.0$ mm ■

6. From Eq. (12.8), $k = \frac{dG}{8NC^3}$ where G = 79 GPa (Appendix C-1)

$$N = \frac{(21.88)(79,000)}{8(8)^3(40)} = 10.55$$ ■

Comment: *A slightly modified solution with d = 22 mm would probably be preferred.

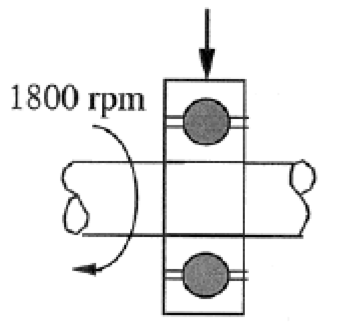
Problem 7

SOLUTION (14.7)

Known: A No. 204 radial ball bearing has a 5000 hr B-10 life at 900 rpm.

Find: Determine the bearing radial load capacity.

Schematic and Given Data:



No. 204 Radial Bearing

$F_r = ?$

90% reliability

Assume: steady loading

B-10 life = 5000 hours

Assumptions:

1. Table 14.2 accurately gives the bearing capacity.
2. Ball bearing life varies inversely with the 10/3 power of the load (i.e., Eq. (14.5a) is accurate).
3. The life given is for a 90% reliability.
4. The loading is steady.

Analysis:

1. From Table 14.1, for a 204 bearing the bore is 20 mm.
2. From Table 14.2, for $L_R = 90 \times 10^6$ rev and a 200 series bearing, C = 3.35 kN.
3. From Fig. 14.13, for 90 percent reliability, $K_r = 1.0$.
4. From Table 14.3, $K_a = 1.0$ for a steady load.
5. From Eq. (14.5a), $L = K_r L_R (C/F_e K_a)^{3.33}$
6. Substituting and solving for F_e: $F_e = F_r = C(L_R/L)^{0.3}$
7. Substituting values:

$$F_r = 3.35\ \text{kN}\left[\frac{90 \times 10^6\ \text{rev}}{(5000\ \text{hr})(60\ \text{min/hr})(900\ \text{rev/min})}\right]^{0.3} = 2409\ \text{N}$$

■

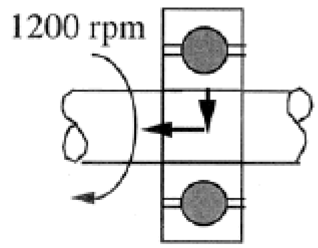
Problem 8

SOLUTION (14.8)

Known: A No. 208 radial ball bearing carries a radial load of 200 lb and a thrust load of 150 lb at 1800 rpm.

Find: Determine the bearing B-10 life.

Schematic and Given Data:



No. 208 Radial Bearing
F_r = 200 N, F_t = 150 N
90% reliability
steady loading
B-10 life = ?

Assumptions:
1. Table 14.2 accurately gives the bearing capacity.
2. Ball bearing life varies inversely with the 10/3 power of the load (i.e., Eq. (14.5a) is accurate).
3. The life given is for a 90% reliability.
4. The load F_e can be found from Eq. (14.3).

Analysis:
1. From Table 14.1, for a 208 bearing the bore is 40 mm.
2. From Table 14.2, for $L_R = 90 \times 10^6$ rev and a 200 series bearing, C = 9.40 kN = 2112.3 lb.
3. From Fig. 14.13, for 90 percent reliability, $K_r = 1.0$.
4. From Table 14.3, $K_a = 1.0$ for a steady load.
5. The ratio $F_t/F_r = 150\text{ lb}/200\text{ lb} = 0.75$
6. The equivalent load from Eq. (14.3) is
$F_e = F_r[1 + 1.115(\{F_t/F_r\} - 0.35)] = 200\text{ lb}[1 + 1.115(\{150/200\} - 0.35)] = 289.2\text{ lb}$
7. From Eq. (14.5a), $L = K_r L_R (C/F_e K_a)^{3.33}$
8. Substituting values into Eq. (14.5a):

$$L = 90 \times 10^6 \text{ rev} \left[\frac{2112.3\text{ lb}}{289.2\text{ lb}}\right]^{3.33} = 6.76 \times 10^{10} \text{ rev}$$

$$= \left[\frac{6.76 \times 10^{10}\text{ rev}}{(60\text{ min/hr})(1200\text{ rev/min})}\right] = 938{,}763\text{ hr}$$

■