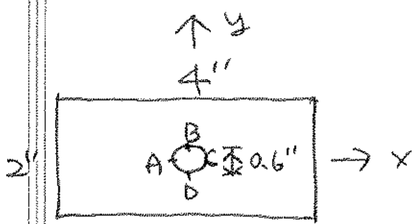
MBD Midterm I solutions - 2009 spring

Problem 1



$$\sigma_x = \sigma_y = \frac{-\alpha(\Delta T)(E)}{1-\alpha}$$

$$= \frac{-(7\times10^{-6})(-200)(30\times10^{6})}{1-0.3}$$

$$= 60{,}000 \ (psi) = 60 \ (ksi)$$

- nominal stress σ_0

at points A and C :

$$\sigma_{0,y} = \sigma_y \left(\frac{4}{4-0.6}\right) = 70.6 \ (ksi)$$

at points B and D :

$$\sigma_{0,x} = \sigma_x \left(\frac{2}{2-0.6}\right) = 85.7 \ (ksi)$$

- stress concentration factor (From Fig 4.40)

at points A and C :

$$\left(\frac{d}{b}\right)_y = \frac{0.6''}{4''} = 0.15 \Rightarrow (K_t)_y = 2.5$$

at points B and D:

$$\left(\frac{d}{b}\right)_x = \frac{0.6''}{2''} = 3 \Rightarrow (K_t)_x = 2.3$$

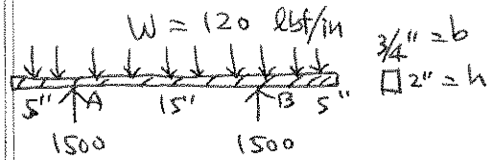
$$\Rightarrow (\sigma_y)_{max} = (\sigma_{0,y})\cdot(K_t)_y = 70.6 \times 2.5 = 176.5 \ (ksi)$$

$$(\sigma_x)_{max} = (\sigma_{0,x})\cdot(K_t)_x = 85.7 \times 2.3 = 197.1 \ (ksi)$$

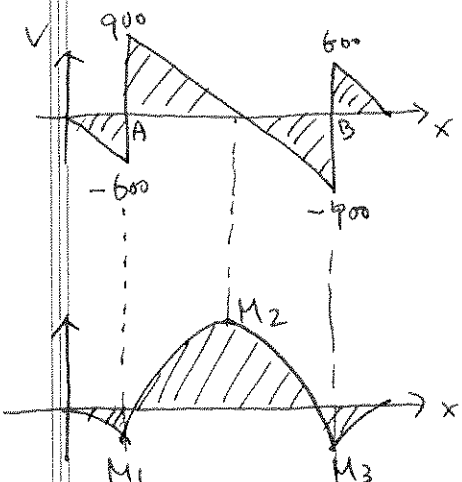
$\Rightarrow$ max stress occurs at B, B with $\sigma_{max} = 197.1$ (ksi)

Problem 2

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(0.75)(2^3) = 0.5 \text{ in}^4$$



(a)



$$V_A^- = -(120)(5) = -600 \text{ lbf}$$

$$V_A^+ = -600 + 1500 = 900 \text{ lbf } \underline{(V_{max})}$$

$$V_B^+ = (120)(5) = 600 \text{ lbf}$$

$$V_B^- = V_B^- - 1500 = -900 \text{ lbf } \underline{(V_{max})}$$

$$M_1 = -\frac{1}{2}(600)(5) = -1500 \text{ (lbf-in)}$$

$$M_2 = -1500 + \frac{1}{2}(900)(7.5) = 1875 \text{ (lbf-in)}$$

$$\Rightarrow M_{max} = M_2 = 1875 \text{ (lbf-in)}$$

(b)

- max tensile bending stress occurs at mid point

$$\sigma_{max} = \frac{M_{max}\left(\frac{h}{2}\right)}{I} = \frac{1875(1)}{0.5} = \underline{3750 \text{ (psi)}}$$

- max transverse shear occurs at A and B

$$\tau = \frac{V Q_{max}}{I t} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{900}{(0.75)(2)} = \underline{900 \text{ (psi)}}$$

Problem 3

$\sigma_x = 30$ ksi , $\sigma_y = -20$ ksi , $\sigma_z = ?$

(a) $\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_3) \le 40$ ksi

Case 1: (max σ_z) $\sigma_z > \sigma_x > \sigma_y$

$$\Rightarrow \frac{\sigma_z - \sigma_y}{2} = 40 \Rightarrow \sigma_z = 80 + \sigma_y = 60 \text{ (ksi)}$$

Case 2: (min. σ_z) $\sigma_x > \sigma_y > \sigma_z$

$$\Rightarrow \frac{\sigma_x - \sigma_z}{2} = 40 \Rightarrow \sigma_z = \sigma_x - 80 = -50 \text{ (ksi)}$$

$\therefore -50 \text{ (ksi)} \le \sigma_z \le 60 \text{ (ksi)}$

(b) $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_x + \sigma_z))$

$\varepsilon_{x,max}$ occurs at $\sigma_z = -50$ ksi

$$\Rightarrow \varepsilon_{x,max} = \frac{1}{30{,}000}(30 - 0.3(-20 - 50)) = 1.7 \times 10^{-3}$$

(c) $\sigma_x = 30$ ksi , $\sigma_y = -20$ ksi , $\sigma_z = -50$ ksi